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Estimation on frequency characteristics of a photodiode determined by the motion of charge carriers in the space charge region on the surface generation of carriers

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Abstract. We have estimated the frequency characteristics of a photodiode determined by the motion of charge carriers in the space-charge region on the surface generation of carriers in two extreme cases of unalloyed and evenly alloyed semiconductors. The expressions allowing the comparison of photodiodes with different constructions have been obtained.

Keywords: photodiode, charge carrier, frequency, photosignal, generation.

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1. Introduction

The frequency characteristics of photodiodes determine their fast operation and ability to register the small-duration pulse signals. These characteristics are mainly determined by the time of flight of photogenerated charge carriers through the the space-charge region (SCR) [1-3]. Therefore, the task to properly estimate these characteristics is actual for the modern pulse optoelectronic technique, in particular for the construction of semiconductor fast-acting *p-i-n* photodiodes.

In works [1, 4-6], separate questions concerning the kinetics of photoresponse of a photodiode on the flight of charge carriers through SCR are considered. However, the results obtained in these works do not allow one to estimate the frequency characteristics of developed *p-i-n* photodiodes in different modes and under different conditions of the generation of charge carriers.

It was shown in [7] that, in the general case of the motion (drift) of charge carriers inside SCR of a semiconductor crystal in the electric field, within the assumptions [1] about the type of modulation of a photosignal, the increase of a current in the external circuit (the current of a photosignal) can be presented as

$$dx\,dx'\,I = \frac{E_x(x)}{\int_x^{x_0} E_x(x)\,dx} \sin\omega\left(t - \int_{x'}^{x} \frac{d\vec{x}''}{\vec{E}_x(x)\bar{\mu}}\right) \frac{\partial I_0}{\partial x'}\, dx'\,dx \tag{1.1}$$

where dx and dx' are the elements of the way passed by a charge carrier in SCR on the distances of x and x', respectively; I – photocurrent; E_x – component of the electric field strength along the x axis; ω – modulation frequency of a photosignal; t – time moment; μ – mobility of charge carriers; I_0 – effective (internal) amplitude of the variable component of the generated photocurrent.

That is, the current in the external circuit depends on two functions of coordinates (the electric field strength and the generation rate of the current of a photosignal), and the value of this current is determined by specific types of these functions.

The extreme cases of practical interest are as follows [1]:

– constant strength of the electric field

$$E_x = E_0 = \text{const}; \tag{1.2}$$

– constant value of volume charge

$$\nabla E_x = \frac{en}{\varepsilon\varepsilon_0}; \tag{1.3}$$

– local generation, including the surface one, of the current of a photosignal

$$\frac{\partial I_0}{\partial x'} = I_0\,\delta(x_0), \tag{1.4}$$

where $\delta(x_0)$ is the ordinary delta-function;

– the generation rate of the current of a photosignal

$$\frac{\partial I_0}{\partial x'} = \frac{I_0}{x_0}, \quad (1.5)$$

which is uniform over the volume.

The purpose of the present work is the analysis of the frequency characteristics and their estimation in the case of the constant strength of the electric field in a crystal and the constant surface generation rate of the current, as well as a constant value of volume charge inside the crystal in the case of the surface generation of the photocurrent.

2. Results and their discussion

2.1. The case of a constant electric field strength in the crystal and the surface generation of a current

Such a situation arises on the absorption of a comparatively short-wave radiation by a photodiode with a very high-resistance base (pure *p-i-n* photodiode). As a result of the complete exhaustion, the high-resistance region becomes negligibly narrow, i.e., conditions (1.2) and (1.4) are satisfied.

Relations (1.1), (1.2), and (1.4) yield

$$dx\,dx'I = \frac{E_x(x)}{\int_x^{x_0} E_x(x)\,dx} \sin\omega\left(t - \int_{x'}^{x} \frac{d\vec{x}''}{\vec{E}_x(x)\overline{\mu}}\right) \times$$

$$\times I_0\delta(x_0')\,dx'dx = \quad (2.1)$$

$$= \frac{E_0}{E_0 x_0} \sin\omega\left(t - \frac{|x - x'|}{E_0\mu}\right) I_0\delta(x_0')\,dx'dx,$$

$$I = \int_x^{x_0} dx' \int_x^{x_0} dx\,\frac{I_0}{x_0}\delta(x_0')\sin\omega\left(t - \frac{|x - x'|}{E_0\mu}\right) =$$

$$= \frac{I_0 E_0 \mu}{x_0\omega}\left[-2\sin\left(t - \frac{|x_0 - x_0'| + |x_0'|}{2E_0\mu}\right)\times\right. \quad (2.2)$$

$$\left.\times\sin\omega\left(-\frac{|x_0 - x_0'| - |x_0'|}{2E_0\mu}\right)\right].$$

In the case of surface generation, it can take two values depending on that which side of the crystal is illumunated:

$$x_0' = \begin{cases} x_0 \\ 0 \end{cases}. \quad (2.3)$$

However, in any of these cases, we have

$$I = \frac{I_0 V_0 \mu}{\omega x_0^2}\sin\frac{\omega x_0^2}{2V_0\mu}\sin\omega\left(t - \frac{x_0^2}{2V_0\mu}\right) \quad (2.4)$$

with regard for relation (2.16) from [1].

Expression (2.4) describes the dependence of the output current of a photosignal in the external circuit on the modulation frequency (ω), structural parameters (x_0 and μ), mode of operation (V_0), and value of the generated current (I_0) [1]

$$I_0 = \int_\nu \frac{\overline{e}_0\gamma(\nu)}{h\nu}\frac{\partial\Phi}{\partial\nu}d\nu, \quad (2.5)$$

i.e., it is determined mainly by the intensity and wavelength of the absorbed radiation. Usually in the comparison of the frequency characteristics of photodetectors, such a parameter as the limiting frequency [see GOST 18952-74] is used:

$$\frac{A(\omega = \omega_{\lim})}{A_{\max}} = \frac{\sqrt{2}}{2}, \quad (2.6)$$

where $A(\omega = \omega_{\lim})$ is the amplitude of the variable component of the output current of a photosignal in the external circuit at the limiting modulation frequency of radiation;

$A_{\max}$ – is the maximal value of the amplitude of the variable component of an output photosignal in the external circuit.

In our case, relation (2.4) yields

$$A_{\max} = A(\omega = 0) = I_0, \quad (2.7)$$

$$\frac{A(\omega = \omega_{\lim})}{A_{\max}} = 2\frac{V_0\mu}{\omega_{\lim} x_0^2}\sin\frac{\omega_{\lim} x_0^2}{2V_0\mu} = \frac{\sqrt{2}}{2}, \quad (2.8)$$

$$\frac{\omega_{\lim} x_0^2}{2V_0\mu} \approx 1.391, \quad (2.9)$$

and the limiting frequency

$$f_{\lim} = \frac{\omega_{\lim}}{2\pi} \approx 1.391\frac{V_0\mu}{\pi x_0^2}. \quad (2.10)$$

Using (2.4), it is always possible to calculate, within the limits of the accepted assumptions, the form of output current signals for the given form of an input signal with the non-sine law of modulation, by using the Fourier transformation (decomposition in a Fourier series). Moreover, with the use of (2.10), it is possible to compare the frequency characteristics of different photodetectors.

2.2. The case of the surface generation of a photocurrent at a constant volume charge inside the crystal

Such a situation arises at the reception of comparatively short-wave radiation by a photodiode with the evenly alloyed base at the bias voltage providing the width of the SCR to be equal exactly to the crystal thickness at a given degree of alloying.

As well as in the previous case, at the even strength of the electric field in the crystal and under the surface generation of current, the total current will be formed by charge carriers of the same sign.

Only the function E_x entering (1.1) will change. From (1.3), we get

$$E_x = \int_0^x \frac{e_0 n}{\varepsilon\varepsilon_0} dx = \frac{e_0 n}{\varepsilon\varepsilon_0} x . \qquad (2.11)$$

Taking relations (1.3), (1.4), and (2.11) into account, we present expression (1.1) as

$$dx\,dx' I = \frac{\frac{e_0 n}{\varepsilon\varepsilon_0} x}{\int_x^{x_0} \frac{e_0 n}{\varepsilon\varepsilon_0} x dx} \times \qquad (2.12)$$

$$\times \sin\omega\left(t - \int_{x'}^{x} \frac{d\vec{x}''}{\frac{e_0 n}{\varepsilon\varepsilon_0} \vec{x}\,\overline{\mu}} \right) I_0 \delta(x_0')\, dx' dx$$

$$\int_0^{x_0} \frac{e_0 n}{\varepsilon\varepsilon_0} x dx = \frac{e_0 n}{\varepsilon\varepsilon_0}\frac{x_0^2}{2} = V_0 = \frac{x_0^2}{2\tau\mu}, \qquad (2.13)$$

$$\int_{x'}^{x} \frac{d\vec{x}''}{\frac{\overline{e_0 n}}{\varepsilon\varepsilon_0} \vec{x}\,\overline{\mu}} = \left| \frac{\varepsilon\varepsilon_0}{e n \mu} \lg\frac{x}{x'} \right|, \qquad (2.14)$$

$$I = \int_x^{x_0} dx' \int_x^{x_0} dx \frac{2x_0}{x_0^2} I\delta(x_0') \sin\omega\left(t - \left| \frac{\varepsilon\varepsilon_0}{en\mu} \lg\frac{x}{x'} \right| \right) =$$

$$= \int_x^{x_0} \frac{2x_0}{x_0^2} x \sin\omega\left(t - \left| \frac{\varepsilon\varepsilon_0}{en\mu} \lg\frac{x}{x'} \right| \right) dx. \qquad (2.15)$$

We now change the variables:

$$t - \left| \frac{\varepsilon\varepsilon_0}{e_0 n\mu} \lg\frac{x}{x'} \right| = u = t - \overline{\tau} \lg\frac{x}{x_0'};$$

$$\frac{\varepsilon\varepsilon_0}{e_0 n\mu} = \tau; \quad t - u = \overline{\tau} \lg\frac{x}{x_0'}; \qquad (2.16)$$

$$\overline{\tau} = \begin{cases} \tau \ \text{at}\ \vec{V} \uparrow\uparrow \vec{x}, \\ -\tau \ \text{at}\ \vec{V} \downarrow\uparrow \vec{x}; \end{cases}$$

$$\lg\frac{x}{x_0'} = \frac{t-u}{\overline{\tau}}; \quad x = x_0' e^{\frac{t-u}{\overline{\tau}}};$$

$$dx = x_0' e^{\frac{t-u}{\overline{\tau}}} \cdot \frac{-du}{\overline{\tau}}. \qquad (2.17)$$

Taking Eqs. (2.15)-(2.17) into account, we obtain

$$I = \int_0^{x_0} \frac{2I_0}{x_0^2} \cdot x_0' \cdot e^{\frac{t-u}{\overline{\tau}}} \cdot \sin\omega u \cdot x_0' \cdot e^{\frac{t-u}{\overline{\tau}}} \cdot \frac{-du}{\overline{\tau}} =$$

$$= 2I_0 \frac{2\sin\omega\left(t - \overline{\tau}\lg\frac{x_0}{x_0'} \right) + \overline{\tau}\,\omega\cos\omega\left(t - \overline{\tau}\lg\frac{x_0}{x_0'} \right)}{4 + \omega^2\tau^2}. \qquad (2.18)$$

This formula becomes

$$I_1 = 2I_0 \frac{2\sin\omega\left(t - \tau\lg\frac{x_0}{x_0'} \right) + \tau\,\omega\cos\omega\left(t - \tau\lg\frac{x_0}{x_0'} \right)}{4 + \omega^2\tau^2} \qquad (2.19)$$

at $x_0' \to 0$ and

$$I_2 = 2 I_0 \frac{2\sin\omega t - \tau\omega\cos\omega t}{4 + \omega^2\tau^2} \qquad (2.20)$$

at $x_0' \to x_0$. The quantity

$$t_0' = \tau \lg\frac{x_0}{x_0'} \qquad (2.21)$$

characterizes the time delay (a change of the phase) of the output signal relative to the input one, which is equal to the time of flight of charge carriers through SCR. According to the idealized formula (2.21), it tends to infinity as $x_0 \to 0$. Really, the time delay is always finite (though it can be large enough), first, due to the accumulation of charge carriers on the SCR border, its expansion, and the presence of thermal motion of charge carriers and atoms and, second, because the region of generation of carriers is always finite really and never tightens into a two-dimensional surface.

It is clear that, in this case, the minimum effective value of x_0 cannot be substantially less than the thickness of the surface layer of atoms. For silicon, it equals 5Å = $5\cdot10^{-10}$ m. Respectively, the maximally possible value of the delay is

$$t_{0\max}' \approx \tau \ln\frac{x_0}{x_0'} \approx \tau \ln\frac{0.5\cdot10^{-3}\,\text{m}}{5\cdot10^{-10}\,\text{m}} = \tau \ln 10^6 \approx 13.8\,\tau \qquad (2.22)$$

for a 500-μm SCR. We now transform (2.18) into

$$I = \frac{4I_0}{4+\omega^2\tau^2}\left[\sin\omega\,(t - \overline{\tau}\ln\frac{x_0}{x_0'}) + \frac{\omega\overline{\tau}}{2}\cos\omega\,(t - \overline{\tau}\ln\frac{x_0}{x_0'}) \right] =$$

$$= I_0 \cdot \varphi\,(\omega,\overline{\tau})\,[\sin\omega\,(t - t_0')\cos\Delta\varphi + \cos\omega\,(t - t_0')\sin\Delta\varphi]. \qquad (2.23)$$

By equating the coefficients of identical terms of the relation, we get

$$\begin{cases} \varphi^2(\omega_1\tau)\cdot\cos\Delta\varphi = \dfrac{1}{1+\dfrac{\omega^2\tau^2}{4}} \\ \varphi(\omega_1\tau)\cdot\sin\Delta\varphi = \dfrac{\dfrac{\omega\overline{\tau}}{2}}{1+\dfrac{\omega^2\tau^2}{4}} \end{cases} \tag{2.24}$$

By squaring all the parts and adding both Eqs, we get

$$\varphi^2(\omega,\tau) = \frac{1+\dfrac{\omega^2\overline{\tau}^2}{4}}{\left(1+\dfrac{\omega^2\tau^2}{4}\right)^2} = \frac{1}{1+\dfrac{\omega^2\tau^2}{4}}, \tag{2.25}$$

$$\varphi(\omega,\tau) = \frac{1}{\sqrt{1+\dfrac{\omega^2\tau^2}{4}}}. \tag{2.26}$$

Taking into account (2.24), we have

$$\cos\Delta\varphi = \frac{1}{\varphi(\omega,\tau)}\cdot\frac{1}{1+\dfrac{\omega^2\tau^2}{4}} = \frac{1}{\sqrt{1+\dfrac{\omega^2\tau^2}{4}}}, \tag{2.27}$$

$$\sin\Delta\varphi = \frac{1}{\varphi(\omega,\tau)}\cdot\frac{\omega\overline{\tau}}{1+\dfrac{\omega^2\tau^2}{4}} = \frac{\omega\overline{\tau}}{\sqrt{1+\dfrac{\omega^2\tau^2}{4}}}, \tag{2.28}$$

$$I = I_0\frac{\sin\left[\omega(t-t_0') + \arccos\dfrac{4}{\sqrt{4+\omega^2\tau^2}}\right]}{\sqrt{1+\dfrac{\omega^2\tau^2}{4}}}. \tag{2.29}$$

Expression (2.29) describes the dependence of the output current of a photosignal in the external circuit on the modulation frequency (ω), structural parameters (t_0', τ'), mode of operation (V_0), whereas the generated current (I_0) is determined mainly by the intensity and the wavelength of the absorbed radiation.

We use (2.6) and (2.8) for the estimation of the limiting frequency:

$$\frac{1}{\sqrt{1+\dfrac{\omega_{\lim}^2\tau^2}{4}}} = \frac{\sqrt{2}}{2}, \tag{2.30}$$

$$\frac{\omega_{\lim}^2\tau^2}{4} = 1, \tag{2.31}$$

$$\omega_{\lim} = \frac{2}{\tau}, \tag{2.32}$$

$$f_{\lim} = \frac{\omega_{\lim}}{2\pi} = \frac{2}{2\pi\tau} = 2\frac{V_0\,\mu}{\pi x_0^2}. \tag{2.33}$$

Using (2.6), it is always possible, within the limits of the accepted assumptions, to calculate the form of the output signal for the given form of an input visual signal with the non-sine law of modulation, using the Fourier transformation. Then relation (2.33) will allow one to compare the frequency characteristics of different photodetectors.

3. Conclusions

1. We have analyzed the frequency characteristics of *p-i-n* photodiodes in the case of a constant strength of the electric field in a crystal with regard for the surface generation of current, as well as for the constant volume charge inside the crystal in the case of the surface generation of a photocurrent.

2. It is shown that the current in the external circuit depends on two functions (on their specific kind) of coordinates (the electric field strength and the generation rate of the current of a photosignal) and is completely determined by the parameters of the material of a photodetector and the absorbed radiation.

3. Simplicity, generality, and exactness of the obtained expressions make them suitable for the usage on the design of new photodiodes.

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